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On the exposure limits for extended source multiple pulse laser exposures

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The proposed revisions to the ANSI Z136, ICNIRP, and IEC 60825-1 laser exposure limits for multiple pulse ocular exposure for wavelengths from 400 to 1400 nm are examined for pulse durations $t_p \geq t_{\min}$ (T_i). The three rules that are defined to be applied for multiple pulse exposures (or for classification for IEC 60825-1) are compared to identify criteria for which one of the rules is the critical one, i.e., the rule that limits the energy per pulse for a given exposure or product emission. Such a comparison can help to simplify a safety analysis, but also guide the design of systems for which the output is to be maximized yet still be classified as a Class 1 or Class 2 laser system. © *2013 Laser Institute of America*.

Key words: laser safety, maximum permissible exposure (MPE), exposure limit, eye, retina, pulse

I. INTRODUCTION

Updates to national and international laser safety standards [American National Standards Institute (ANSI) Z136.1 and International Electrotechnical Commission (IEC) 60825-1, respectively], and of the International Commission on NonIonizing Radiation Protection (ICNIRP) laser exposure limit guidelines, are expected to be published in 2013. These standards provide guidance for determining the maximum permissible exposure (MPE) to laser irradiation. The user of these standards must consider the wavelength of the radiation emitted by the laser or laser system, the duration of the exposure, and in the retinal hazard wavelength region (400 to 1400 nm), the angular subtense of the source. For a multiple pulse or repetitive-pulse exposure, these factors interact, in part because the user must compare the intended exposure to the MPE following each of three rules to fully ensure the safety of the exposure:

- Rule 1: The exposure from any single pulse of duration t_p in a sequence of pulses shall not exceed the MPE for a single pulse of duration t_p.
- Rule 2: The exposure from any group of pulses delivered in time T shall not exceed the MPE for an exposure of duration T.
- Rule 3: The exposure for any single pulse ($t_p < 0.25$ s) shall not exceed the MPE based on the width of a single pulse multiplied by a multiple pulse correction factor C_P (C_5). Note that for pulse durations less than $t_{\min}$ (T_1), special rules apply.

For pulsed exposure to a sequence of identical pulses at a constant pulse frequency, current standards[1–3] only require consideration of the pulse duration (t_p), the maximum considered exposure duration ($T_{\max}$), and the number of pulses within any duration $T \leq T_{\max}$ in order to compute the MPE by each of the three rules. The correction factor C_E for the angular subtense of the source is independent of the exposure duration and therefore affects the MPE via each of the three rules equally. Thus, it is relatively straight-forward to determine which of the three rules is most restrictive for a given exposure. However, changes proposed for the next editions of ANSI Z136.1 and IEC 60825-1 will cause $C_E(C_6)$ to be dependent on the pulse and total exposure durations. In addition, the definition of the multiple pulse correction factor $C_P(C_5)$ will be modified. As a result, the process for comparing the MPE for a multiple pulse exposure by each of the three rules will be less intuitive.

This paper presents an analysis of the proposed exposure limits for multiple pulse laser exposure to the eye for wavelengths in the visible and near infra-red (400 to 1400 nm). Only the limits to protect against thermally induced damage to the retina are considered. Limits to protect against photochemically induced retinal injury, which become important for longer duration exposures and shorter wavelengths, and corneal limits, which become important for longer wavelengths (from about 1200 to 1400 nm onwards) are not considered. Furthermore, the analysis is limited to exposures to a sequence of identical, equally spaced pulses, i.e., each pulse has the same duration t_p, and energy Q_e (or equivalently, corneal radiant exposure H_e). Furthermore, only pulse durations $t_p > t_{\min}$ are considered. Since the accessible emission limits for Class 1, Class 2, and Class 3 R are directly derived from the ocular exposure limits in the ANSI and IEC laser safety standards, the discussion also relates in a similar manner to the classification of laser products.

The main goal of this paper is to identify and discuss criteria that can identify which of the multiple pulse exposure rules provides the most restrictive exposure or emission limit depending on the source size, pulse repetition frequency, pulse duration, and total exposure duration. These criteria can help to simplify a safety analysis by identifying which one of the three rules needs to be applied. The analysis may also guide the optimization of the output of some products

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TABLE I. Symbols used in this paper.

t_p	Pulse duration (s)
$T_{\max}$	Maximum assumed exposure duration (s) for which MPE analysis is performed (typically 0.25 or 10 s)
T, t	Any exposure duration $\leq T_{\max}$
N	Number of pulses occurring in time T
F	Pulse repetition frequency (pulses/s)
$F_{\text{cross}}^{1\to 2}$	Crossover pulse frequency for determining when rule 1 or rule 2 provides the more restrictive exposure or emission limit
$F_{\text{cross}}^{3\to 2}$	Crossover pulse frequency for determining when rule 3 or rule 2 provides the more restrictive exposure or emission limit
$\mathcal{D}$	Duty cycle
C_A, C_C	Wavelength-dependent correction factors
C_E	Extended source correction factor
C_P	Multiple pulse correction factor
α	Apparent source angular subtense (mrad)
t_α	Pulse duration for which $\alpha_{\max} = \alpha$
$t_{\min}$	Shortest exposure duration for which the exposure limit, expressed as radiant exposure, is proportional to $t^{3/4}$ ("thermal regime")
T_2	Exposure duration beyond which extended source limits are expressed as constant irradiance
H_{pulse}	Corneal radiant exposure from a single pulse of duration t_p
MPE(t)	Maximum permissible exposure limit for duration t second, expressed as corneal radiant exposure

which must still be capable of being classified as Class 1 or Class 2 laser products.

Symbols used in this paper are listed in Table I, and are taken primarily from the ANSI Z136.1 (Ref. 1) guideline. Symbols for equivalent parameters for the ICNIRP (Ref. 2) and IEC 60825-1 (Ref. 3) exposure limits are listed in Table II.

Note that the exposure duration T, the number of pulses N, and the pulse repetition frequency F are related through the equation $N = F \times T$ or $F = N/T$. The duty cycle, the fraction of the time, the pulse is "on," is given by $\mathcal{D} = (N \times t_p)/T = t_p \times F$.

In the proposed revisions to the multiple pulse exposure limits, the multiple pulse correction factor C_P (C_5) in the ANSI standard differs from 1.0 only for single-pulse durations $t_p > t_{\min}$, where

$$t_{\min} = \begin{cases} 5\,\mu\text{s} & \text{for} \quad 400\,\text{nm} \leq \lambda < 1050\,\text{nm} \\ 13\,\mu\text{s} & \text{for} \quad 1050\,\text{nm} \leq \lambda < 1400\,\text{nm} \end{cases}. \tag{1}$$

The revised C_P is shown in Fig. 1. In the draft ICNIRP guidelines and IEC laser safety standards, for exposure durations less than $t_{\min}$ (where the injury mechanism is microcavitation around melanosomes), the rules differ from the draft ANSI standard, as a reduction factor is defined which is applied to the single-pulse limits for the case of more than 600 pulses and exposure durations of longer than 0.25 s. The analysis here will only deal with pulse durations in the thermal regime, i.e., individual pulse durations longer than $t_{\min}$. Since $t_p > 0.25$ s is considered a continuous wave exposure, the analysis will be for pulse durations in the range $t_{\min} \leq t_p < 0.25$ s. Results will be applicable for $t_{\min} < T < T_{\max}$, and specific calculations for $T_{\max} = 0.25$ s, and $T_{\max} = 10$ s will be given.

TABLE II. Symbols for equivalent parameters in the ANSI Z136.1, ICNIRP, and IEC 60825-1 laser exposure guidelines.

ANSI Z136.1	ICNIRP	IEC 60825-1
C_A	C_A	C_4
C_C	C_C	C_7
C_E	C_E	C_6
C_P	C_P	C_5
$t_{\min}$	t_i	T_i
T_2	T_2	T_2
α	α	α
MPE	EL	MPE

- If $t_p \leq t_{\min}$:
  - (ANSI Z136) $C_P = 1.0$
  - (ICNIRP, IEC 60825-1) $C_P = 1.0$

    If $T > 0.25$ s and $N > 600$ pulses, then $C_P = 5N^{-1/4}$
- If $t_p > t_{\min}$:
  - For $\alpha \leq 5$ mrad, $C_P = 1.0$
  - For 5 mrad $< \alpha \leq \alpha_{\max}(t_p)$

    $$C_P = \begin{cases} N^{-1/4} & \text{for} \quad N \leq 40 \\ 0.4 & \text{for} \quad N > 40 \end{cases}$$
  - For $\alpha_{\max}(t_p) < \alpha \leq 100$ mrad,

    $$C_P = \begin{cases} N^{-1/4} & \text{for} \quad N \leq 625 \\ 0.2 & \text{for} \quad N > 625 \end{cases}$$
  - For $\alpha > 100$ mrad, $C_P = 1$

FIG. 1. Revised multiple pulse correction factor C_P (C_5).

II. EXPOSURE LIMITS FOR 400 TO 1400 NM

A. Wavelength-dependent correction factors

The wavelength-dependent correction factor C_A has not been changed from the form given in ANSI Z136.1-2007 (Ref. 1):

$$C_A = \begin{cases} 1.0 & \text{for} \quad 400\,\text{nm} \le \lambda < 700\,\text{nm} \\ 10^{0.002(\lambda-700)} & \text{for} \quad 700\,\text{nm} \le \lambda < 1050\,\text{nm} \\ 5.0 & \text{for} \quad 1050\,\text{nm} \le \lambda < 1400\,\text{nm} \end{cases} . \tag{2}$$

In this expression, the wavelength λ must be in nanometers.

The wavelength-dependent factor C_C has undergone significant revision for wavelengths from 1200 to 1400 nm, leading to substantial increase in the exposure limits for this range. For a discussion of the rationale for this change, see Zuclich *et al.*[4] In order to facilitate the analysis, the definition of the wavelength correction factor C_C is extended to wavelengths less than 1050 nm in an obvious manner

$$C_C = \begin{cases} 1.0 & \text{for} \quad 400\,\text{nm} \le \lambda < 1150\,\text{nm} \\ 10^{0.018(\lambda-1150)} & \text{for} \quad 1150\,\text{nm} \le \lambda < 1200\,\text{nm} \\ 8 + 10^{0.04(\lambda-1250)} & \text{for} \quad 1200\,\text{nm} \le \lambda < 1400\,\text{nm} \end{cases} , \tag{3}$$

where λ is in nanometers.

B. Source extent correction factors

In the proposed revisions to the extended source exposure limits, the angular subtense α_{max} is now no longer a constant value of 100 mrad for retinal thermal effects,[5] but depends on the exposure duration:

$$\alpha_{max}(t) = \begin{cases} 5\,\text{mrad} & \text{for} \quad t < 625\,\mu\text{s} \\ 200\sqrt{t}\,\text{mrad} & \text{for} \quad 625\,\mu\text{s} \le t < 0.25\,\text{s} \\ 100\,\text{mrad} & \text{for} \quad t \ge 0.25\,\text{s} \end{cases} , \tag{4}$$

where t is in seconds. For source angular subtense from 5 to 100 mrad, there will be a value of the exposure duration, t_α, for which $\alpha_{max}(t_\alpha) = \alpha$. From the second row of Eq. (4), the value for t_α is given by

$$t_\alpha(\text{s}) = \left[\frac{\alpha(\text{mrad})}{200}\right]^2, \tag{5}$$

where $625\,\mu\text{s} \le t_\alpha < 0.25\,\text{s}$. Note that $\alpha > \alpha_{max}(t)$ if $t < t_\alpha$, while $\alpha < \alpha_{max}(t)$ for $t > t_\alpha$.

Because of this time-dependence of α_{max}, the source extent correction factor C_E is now a function not only of the source angular subtense α, but also on the exposure (or pulse) duration t. To emphasize this, the source extent correction factor will hereafter be written as

$$C_E(\alpha, t) = \begin{cases} 1.0 & \text{for} \quad \alpha < \alpha_{min} \\ \alpha/\alpha_{min} & \text{for} \quad \alpha_{min} \le \alpha < \alpha_{max}(t) \\ \alpha^2/(\alpha_{min} \cdot \alpha_{max}(t)) & \text{for} \quad \alpha \ge \alpha_{max}(t) \end{cases} , \tag{6}$$

where $\alpha_{min} = 1.5$ mrad. Note that this form for C_E for $\alpha > \alpha_{max}$ applies for a source that is homogeneous and circular. The determination of the exposure level can then be done with a nonrestricted field of view, and then C_E increases with the square of α for $\alpha > \alpha_{max}$. For an inhomogeneous or non-circular source profile, a more complicated analysis using a varying field of view is required. The reader is referred to Henderson and Schulmeister[6] for further discussion.

For source angular subtense $\alpha > 5$ mrad, C_E can be written without explicit reference to α_{max}. Using Eqs. (4) and (5) in Eq. (6):

- If $\alpha < \alpha_{min}$,

$$C_E(\alpha, t) = 1.0. \tag{7a}$$

- If $\alpha_{min} \le \alpha < 5$ mrad,

$$C_E(\alpha, t) = \alpha/\alpha_{min}. \tag{7b}$$

- If $5\,\text{mrad} \le \alpha < 100\,\text{mrad}$,

$$C_E(\alpha, t) = \begin{cases} \alpha^2/7.5 & \text{for} \quad t < 625\,\mu\text{s} \\ (\alpha/\alpha_{min})\sqrt{t_\alpha/t} & \text{for} \quad 625\,\mu\text{s} \le t \le t_\alpha \\ \alpha/\alpha_{min} & \text{for} \quad t > t_\alpha \end{cases} . \tag{7c}$$

- If $\alpha \ge 100$ mrad,

$$C_E(\alpha, t) = \begin{cases} \alpha^2/7.5 & \text{for} \quad t < 625\,\mu\text{s} \\ (\alpha/\alpha_{min})\sqrt{t_\alpha/t} & \text{for} \quad 625\,\mu\text{s} \le t \le 0.25\,\text{s} \\ \alpha^2/150 & \text{for} \quad t > 0.25\,\text{s} \end{cases} . \tag{7d}$$

C. MPE

By extending the definition of C_C to wavelengths below 1050 nm (Eq. (3)), the maximum permissible exposure to protect against thermal retinal damage for exposure durations $t > t_{min}$ can be succinctly expressed for all wavelengths from 400 to 1400 nm. The revised MPE to be promulgated in the updated ANSI, ICNIRP, and IEC laser exposure guidelines is given by

$$\text{MPE}(t > t_{min}) = \begin{cases} 1.8 \times 10^{-3}\, C_A\, C_C\, C_E(\alpha, t)\, t^{3/4}\,\text{J cm}^{-2} \\ \qquad \text{for } t_{min} \le t < T_2 \\ 1.8 \times 10^{-3}\, C_A\, C_C\, C_E(\alpha, t)\, t\, T_2^{-1/4}\,\text{J cm}^{-2} \\ \qquad \text{for } t \ge T_2 \end{cases} , \tag{8}$$

where $T_2 = 10 \times 10^{(\alpha-1.5)/98.5}$ s. In Eq. (8), the limit for exposure durations longer than T_2, which is expressed as a

constant irradiance value in the standards, is expressed here as a radiant exposure, obtained by multiplying by the exposure duration t. Note that the minimum value of T_2 is 10 s; therefore, $t_p < T_2$ for all cases.

D. Multiple pulse correction factor

In the proposed exposure limit revisions, the multiple pulse correction factor C_P has been revised extensively. The new form for C_P is shown in Fig. 1. Note that for collimated beams and other small sources and according to ANSI also for q-switched lasers, there is no longer a multiple pulse correction factor, i.e., $C_P = 1$. This will greatly simplify the hazard evaluation for most laser systems of practical interest.

For pulse duration $t_p > t_{\min}$, $C_P \neq 1$ only for source angular subtense in the range $5\,\text{mrad} < \alpha \leq 100\,\text{mrad}$. In this case, the definition of C_P can be rewritten in terms of the pulse duration without explicit reference to $\alpha_{\max}$. Using the definition of t_α from Eq. (5), and recalling that $\alpha \leq \alpha_{\max}$ for $t_p \geq t_\alpha$:

- If $5\,\text{mrad} < \alpha \leq 100\,\text{mrad}$
  - For $t_{\min} < t_p < t_\alpha$,

$$C_P = \begin{cases} N^{-1/4} & \text{for} \quad N \leq 625 \\ 0.2 & \text{for} \quad N > 625 \end{cases}. \tag{9a}$$

  - For $t_p \geq t_\alpha$,

$$C_P = \begin{cases} N^{-1/4} & \text{for} \quad N \leq 40 \\ 0.4 & \text{for} \quad N > 40 \end{cases}. \tag{9b}$$

- Otherwise, $C_P = 1$.

E. Multiple pulse exposure limit rules

The procedure for determining the exposure or emission limit for a multiple pulse exposure will, in principle, be the same as described in Sec. 8.2.3 of ANSI Z136.1-2007 (Ref. 1) and in Sec. 8.3f in IEC 60825-1:2007,[3] although the revised MPE (Eq. (8)) will be used. The procedure is given as a set of three rules. A full explanation and rationale for the rules will not be given here, but can, for instance, be found in Chap. 3 of Henderson and Schulmeister.[6] Here, the rules for a sequence of identical, equally spaced pulses, i.e., constant rate, are summarized symbolically as follows (for $t_p > t_{\min}$):

Rule 1: Single-pulse MPE: $H_{\text{pulse}} \leq \text{MPE}(t_p)$

Since $t_p < T_2$, rule 1 states that

$$\begin{aligned} H_{\text{pulse}} &< \text{MPE}(t_p) \\ &= 1.8 \times 10^{-3}\, C_A C_C\, C_E(\alpha, t_p)\, {t_p}^{3/4}\,\text{J cm}^{-2}. \end{aligned} \tag{10}$$

Rule 2: Average power MPE: $H_{\text{pulse}} \leq \text{MPE}(T)/N$

Rule 2 states that the single-pulse radiant exposure must satisfy

$$\begin{aligned} H_{\text{pulse}} &< \text{MPE}(T)/N \\ &= \begin{cases} 1.8 \times 10^{-3}\, C_A\, C_C\, C_E(\alpha, T)\, T^{3/4} N^{-1}\,\text{J cm}^{-2} \\ \qquad \text{for } 0.25\,\text{s} \leq T < T_2 \\ 1.8 \times 10^{-3}\, C_A\, C_C\, C_E(\alpha, T) T_2^{-1/4}\, T N^{-1}\,\text{J cm}^{-2} \\ \qquad \text{for } T > T_2 \end{cases}. \end{aligned} \tag{11}$$

Rule 3: Multiple pulse MPE for thermal hazard: $H_{\text{pulse}} \leq C_P \times \text{MPE}(t_p)$

Rule 3 requires

$$\begin{aligned} H_{\text{pulse}} &< C_P \cdot \text{MPE}(t_p) \\ &= 1.8 \times 10^{-3}\, C_A\, C_C\, C_E(\alpha, t_p)\, C_P\, {t_p}^{3/4}\,\text{J cm}^{-2}. \end{aligned} \tag{12}$$

In general, where the pulse rate, pulse energy, and pulse duration may not be constant during the total exposure duration $T_{\max}$, subgroups of pulses of all durations $T < T_{\max}$ must also be examined using the three rules to determine the most limiting case.

III. BASIC ANALYSIS OF MULTIPLE PULSE RULES

The analysis presented in this paper is for a sequence of identical pulses. The "average power" rule then becomes a comparison of the radiant exposure from a single pulse to a *per pulse* MPE determined by dividing the MPE for the duration of the sequence of pulses, T, by the total number of pulses in the train. In the current editions of the laser safety standards,[1–3] within the retinal thermal hazard region $t > 18\,\mu\text{s}$ (for wavelengths from 400 to 1050 nm) or $t > 50\,\mu\text{s}$ (for wavelengths from 1050 to 1400 nm), rule 3 is always applied, and $\alpha_{\max}$ is a constant value of 100 mrad. In this case, it is simple to determine that rule 3 is always the most critical one of the three rules as long as the pulse repetition rate is less than a "critical frequency" of 55.5 kHz (i.e., $1/18\,\mu\text{s}$) for wavelengths from 400 to 1050 nm, or 20 kHz for wavelengths from 1050 to 1400 nm, while rule 2 is the critical rule for higher repetition rates. (In fact, rule 3 becomes equal to rule 2 for pulse rates above the critical frequency within current exposure guidelines.[1–3])

For the proposed revisions to the standards, in which $\alpha_{\max}$ for extended sources depends on the pulse duration, there are no simple criteria to determine which one of the three rules is the critical one, that is, limits the exposure level or limits the permitted output of the device for a given safety class. For instance, for cases where C_P equals unity, whether it is the single-pulse or the average power rule, the more restrictive one depends on the pulse repetition frequency. For cases in which $C_P < 1$, the MPE derived from rule 3 will be more restrictive than the MPE determined from rule 1 for the laser pulse trains considered here, and then the question is if rule 3 is more restrictive than rule 2. We therefore seek the answer to the two questions:

- Under what conditions is the MPE determined by rule 1 more restrictive than the MPE determined by rule 2, the "average power" rule?

- Under what conditions is the MPE determined by rule 3 more restrictive than the rule 2 MPE?

A. When is rule 1 more restrictive than rule 2?

The question of whether rule 1 is more restrictive than rule 2 applies for cases in which $C_P = 1$, i.e., for $\alpha \leq 5$ mrad and $\alpha \geq 100$ mrad. The general analysis will also serve as the basis for the comparison of rules 3 and 2 presented in Sec. III B.

According to Eqs. (10) and (11), rule 1 will be more restrictive than rule 2 if

$$\mathrm{MPE}(t_p) < \mathrm{MPE}(T)/N. \tag{13}$$

1. $T \leq T_2$

For $T \leq T_2$, the condition of Eq. (13) becomes

$$1.8\, C_A\, C_C\, C_E(\alpha, t_p)\, t_p^{3/4} < 1.8\, C_A\, C_C\, C_E(\alpha, T)\, T^{3/4}\, N^{-1}. \tag{14}$$

Canceling common terms on both sides reduces this equation to

$$C_E(\alpha, t_p)\, t_p^{3/4} < C_E(\alpha, T)\, T^{3/4}\, N^{-1} \tag{15}$$

or, on rearranging

$$\left[\frac{N^4\, t_p^3}{T^3}\right]^{1/4} < \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{16}$$

2. $T > T_2$

For $T > T_2$, the condition of Eq. (13) becomes

$$1.8\, C_A\, C_C\, C_E(\alpha, t_p)\, t_p^{3/4} < 1.8\, C_A\, C_C\, C_E(\alpha, T)\, T_2^{-1/4}\, T\, N^{-1}. \tag{17}$$

On canceling common terms and rearranging, this becomes

$$\left[\frac{N^4\, t_p^3\, T_2}{T^4}\right]^{1/4} < \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{18}$$

3. Discussion

To simplify the form of the above equations, it is convenient to define a factor $\mathcal{Z}$ to be equal to the quantity in the square brackets in Eqs. (16) and (18):

$$\mathcal{Z} = \begin{cases} t_p{}^3\, N^4/T^3 & \text{for} \quad T \leq T_2 \\ t_p{}^3\, N^4\, T_2/T^4 & \text{for} \quad T > T_2 \end{cases}. \tag{19}$$

Then the conditions determining if rule 1 provides a more restrictive exposure limit than rule 2 (Eqs. (16) and (18)) can be compactly written as

$$\mathcal{Z}^{1/4} < \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{20}$$

The ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ contains all explicit dependence on the source extent α, and is implicitly dependent on the pulse duration t_p and exposure duration T through $\alpha_{\max}(t_p)$ and $\alpha_{\max}(T)$. Explicit dependence on the pulse duration, total exposure duration, and number of pulses is contained in $\mathcal{Z}$. By using the definitions of the pulse repetition frequency ($F = N/T$) and the duty cycle ($\mathcal{D} = Ft_p = Nt_p/T$), the quantity $\mathcal{Z}$ may be written in several forms:

$$\mathcal{Z} = \begin{cases} t_p{}^3\, F^4\, T & \text{for} \quad T \leq T_2 \\ t_p{}^3\, F^4\, T_2 & \text{for} \quad T > T_2 \end{cases}, \tag{21}$$

$$\mathcal{Z} = \begin{cases} \mathcal{D}^3 FT & \text{for} \quad T \leq T_2 \\ \mathcal{D}^3 FT_2 & \text{for} \quad T > T_2 \end{cases}, \tag{22}$$

$$\mathcal{Z} = \begin{cases} \mathcal{D}^3 N & \text{for} \quad T \leq T_2 \\ \mathcal{D}^3 N(T_2/T) & \text{for} \quad T > T_2 \end{cases}. \tag{23}$$

The form to use for $\mathcal{Z}$ depends on which combination of the parameters pulse duration (t_p), exposure duration (T), number of pulses (N), pulse repetition frequency (F), or duty cycle ($\mathcal{D}$) are available or are of primary interest. Since it is the pulse repetition frequency F that is often of primary interest, the critical frequency for a selection of typical exposure durations will be discussed in Secs. IV and V.

B. When is rule 3 more restrictive than rule 2?

According to Eqs. (11) and (12), rule 3 will provide the lower exposure limit if

$$C_P \cdot \mathrm{MPE}(t_p) < \mathrm{MPE}(T)/N. \tag{24}$$

Comparing this with Eq. (13), the only difference is the factor C_P multiplying the left side of the inequality. The analysis will be similar to that leading to Eq. (20) if the left side of all inequalities is multiplied by C_P. Rule 3 will, therefore, yield a more restrictive, i.e., lower MPE, than rule 1 if

$$C_P \cdot \mathcal{Z}^{1/4} < \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{25}$$

C. *The ratio* $C_E(\alpha,T)/C_E(\alpha,t_p)$

In Eqs. (20) and (25), effects of the source size are contained in the ratio of the source extent correction factors, $C_E(\alpha,T)/C_E(\alpha,t_p)$. To calculate this ratio, four ranges for the source angular subtense α must be considered:

- $\alpha < \alpha_{\min}$
  In this range, $C_E(\alpha, T) = C_E(\alpha, t_p) = 1$ (see Eq. (6)). Therefore, the ratio of these quantities is equal to 1.
- $\alpha_{\min} \leq \alpha < \alpha_{\max}(t_p)$
  In this range, $C_E(\alpha, T) = C_E(\alpha, t_p) = \alpha/\alpha_{\min}$. Therefore, the ratio is once again equal to 1.

- $\alpha_{\max}(t_p) \leq \alpha < \alpha_{\max}(T_{\max})$
  In this range, $C_E(\alpha, T) = \alpha/\alpha_{\min}$, while $C_E(\alpha, t_p) = \alpha^2/(\alpha_{\min} \cdot \alpha_{\max}(t_p))$. Therefore, the ratio $C_E(\alpha, T)/C_E(\alpha, t_p) = \alpha_{\max}(t_p)/\alpha$.
- $\alpha \geq \alpha_{\max}(T)$
  In this range, $C_E(\alpha, T) = \alpha^2/(\alpha_{\min} \cdot \alpha_{\max}(T))$ and $C_E(\alpha, t_p) = \alpha^2/(\alpha_{\min} \cdot \alpha_{\max}(t_p))$. Therefore, the ratio $C_E(\alpha, T)/C_E(\alpha, t_p) = \alpha_{\max}(t_p)/\alpha_{\max}(T)$.
  To summarize,

$$C_E(\alpha, T)/C_E(\alpha, t_p) = \begin{cases} 1.0 & \text{for} \quad \alpha < \alpha_{\max}(t_p) \\ \alpha_{\max}(t_p)/\alpha & \text{for} \quad \alpha_{\max}(t_p) \leq \alpha < \alpha_{\max}(T) \\ \alpha_{\max}(t_p)/\alpha_{\max}(T) & \text{for} \quad \alpha \geq \alpha_{\max}(T) \end{cases}. \tag{26}$$

To examine Eq. (26) in detail, we consider three ranges for the source angular subtense α. In the following, $t_{\min} < t_p < T$ and $t_p < 0.25$ s.

- $\alpha < 5$ mrad
  For source subtense in this range, $\alpha < \alpha_{\max}(t_p)$ for all values of the pulse duration $t_p > t_{\min}$. Therefore, from Eq. (26),

$$C_E(\alpha, T)/C_E(\alpha, t_p) = 1. \tag{27}$$

- 5 mrad $\leq \alpha <$ 100 mrad
  Here, we must consider the exposure duration T as well as the pulse duration t_p.
  - $T < 625\ \mu$s: For exposure durations this short, both $\alpha_{\max}(T)$ and $\alpha_{\max}(t_p)$ are equal to 5 mrad. In this subtense range, therefore, $\alpha \geq \alpha_{\max}(T)$. From the third condition of Eq. (26),

$$C_E(\alpha, T < 625\ \mu\text{s})/C_E(\alpha, t_p) = 1. \tag{28a}$$

  - $T > 0.25$ s: For longer exposure durations, $\alpha_{\max}(T) = 100$ mrad. Therefore, $\alpha < \alpha_{\max}(T)$ in this subtense range. If $t_p \leq t_\alpha$ (Eq. (5)), then $\alpha \geq \alpha_{\max}(t_p)$, and the second condition of Eq. (26) applies. If $t_p > t_\alpha$, then $\alpha < \alpha_{\max}(t_p)$, and the first condition of Eq. (26) applies. We therefore have

$$C_E(\alpha, T > 0.25\,\text{s})/C_E(\alpha, t_p) = \begin{cases} 5/\alpha & \text{for} \quad t_p < 625\ \mu\text{s} \\ 200\sqrt{t_p}/\alpha = \sqrt{t_p/t_\alpha} & \text{for} \quad 625\ \mu\text{s} \leq t_p \leq t_\alpha \\ 1 & \text{for} \quad t_p > t_\alpha \end{cases}. \tag{28b}$$

  - $625\ \mu\text{s} \leq T \leq 0.25$ s: Here, T must be compared to t_α. If $T \leq t_\alpha$, then $\alpha \geq \alpha_{\max}(T)$, and the third condition of Eq. (26) applies with $\alpha_{\max}(T) = 200\sqrt{T}$ mrad:

$$C_E(\alpha, 625\ \mu\text{s} < T \leq t_\alpha)/C_E(\alpha, t_p) = \begin{cases} 5/200\sqrt{T} = 1/40\sqrt{T} & \text{for} \quad t_p < 625\ \mu\text{s} \\ 200\sqrt{t_p}/200\sqrt{T} = \sqrt{t_p/T} & \text{for} \quad 625\ \mu\text{s} \leq t_p \leq t_\alpha \end{cases}. \tag{28c}$$

  If $T > t_\alpha$, then $\alpha < \alpha_{\max}(T)$, and the results are identical to the case for $T > 0.25$ s obtained above

$$C_E(\alpha, T > t_\alpha)/C_E(\alpha, t_p) = \begin{cases} 5/\alpha & \text{for} \quad t_p < 625\ \mu\text{s} \\ \sqrt{t_p/t_\alpha} & \text{for} \quad 625\ \mu\text{s} \leq t_p \leq t_\alpha \\ 1 & \text{for} \quad t_p > t_\alpha \end{cases}. \tag{28d}$$

- $\alpha \geq 100$ mrad
  In this subtense range, $\alpha \geq \alpha_{\max}(T)$ for all exposure durations T. $C_E(\alpha,T)/C_E(\alpha,t_p)$ is thus equal to $\alpha_{\max}(t_p)/\alpha_{\max}(T)$. For $T \leq 0.25$ s, we obtain the same result as that derived above for 5 mrad $\leq \alpha <$ 100 mrad for the case where $\alpha \geq \alpha_{\max}$. For $T > 0.25$ s, $\alpha_{\max}(T) = 100$ mrad, and

$$C_E(\alpha, T)/C_E(\alpha, t_p) = \alpha_{\max}(t_p)/100. \tag{29}$$

The results for the ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ for pulse duration $t_p > t_{\min}$ are summarized in Fig. 2.

The dependence of the ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ on the pulse duration t_p for several values of the source angular subtense α is shown in Fig. 3 for $T \geq 0.25\ \mu$s. Note that, for a given value of α, $C_E(\alpha,T)/C_E(\alpha,t_p)$ reaches the value of 1.0 at the pulse duration t_α.

Figure 4 shows the dependence of $C_E(\alpha,T)/C_E(\alpha,t_p)$ on the source subtense for selected values of the pulse duration t_p. For pulse durations less than 0.25 s, the ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ begins to decrease from a value of 1.0 at an angular subtense equal to $200\sqrt{t_p}$ mrad, and then falls off as α^{-1} until, at $\alpha = 100$ mrad, the ratio is equal to $2\sqrt{t_p}$.

IV. CRITERIA FOR RULE 1 < RULE 2

If the total exposure duration $T_{\max}$, pulse duration t_p, and source extent α are known, an upper limit on the pulse repetition frequency for which rule 1 yields a lower exposure limit than rule 2 is found by using the form for $\mathcal{Z}$ given by Eq. (21) in Eq. (20). For exposure durations $T \leq T_2$, this reduces to

$$F < (t_p^3\, T)^{-1/4} \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{30}$$

The right-hand side of this inequality is labelled the "crossover" pulse repetition frequency for the comparison of the rule 1 and rule 2 limits:

$$F_{\text{cross}}^{1\to 2} = (t_p^3\, T)^{-1/4} \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{31}$$

When $F < F_{\text{cross}}^{1\to 2}$, the single-pulse MPE determined from rule 1 will be more restrictive than the MPE determined from rule 2, the average power rule. When $F > F_{\text{cross}}^{1\to 2}$, the exposure limit determined by the average power rule, rule 2, will be lower than the single-pulse rule limit.

An upper limit on the number of pulses within T can be obtained using $N = F \times T$:

- For $\alpha < 5\,\mathrm{mrad}$

$$C_E(\alpha, T)/C_E(\alpha, t_p) = 1$$

- For $5\,\mathrm{mrad} \le \alpha < 100\,\mathrm{mrad}$

  – $T < 625\,\mu\mathrm{s}$:

$$C_E(\alpha, T)/C_E(\alpha, t_p) = 1$$

  – $625\,\mu\mathrm{s} \le T \le t_\alpha$:

$$\frac{C_E(\alpha,T)}{C_E(\alpha,t_p)} = \begin{cases} 1/40\sqrt{T} & \text{for} \quad t_p < 625\,\mu\mathrm{s} \\ \sqrt{t_p/T} & \text{for} \quad 625\,\mu\mathrm{s} \le t_p < T \end{cases}$$

  – $T > t_\alpha$:

$$\frac{C_E(\alpha,T)}{C_E(\alpha,t_p)} = \begin{cases} 5/\alpha & \text{for} \quad t_p < 625\,\mu\mathrm{s} \\ \sqrt{t_p/t_\alpha} & \text{for} \quad 625\,\mu\mathrm{s} \le t_p \le t_\alpha \\ 1 & \text{for} \quad t_\alpha < t_p < T \end{cases}$$

  Note: $t_\alpha = (\alpha/200)^2$, where t_α is in s, α is in mrad.

- For $\alpha \ge 100\,\mathrm{mrad}$

  – $T < 625\,\mu\mathrm{s}$:

$$C_E(\alpha, T)/C_E(\alpha, t_p) = 1$$

  – $625\,\mu\mathrm{s} \le T < 025\,\mathrm{s}$:

$$\frac{C_E(\alpha,T)}{C_E(\alpha,t_p)} = \begin{cases} 1/40\sqrt{T} & \text{for} \quad t_p < 625\,\mu\mathrm{s} \\ \sqrt{t_p/T} & \text{for} \quad 625\,\mu\mathrm{s} \le t_p < 0.25\,\mathrm{s} \end{cases}$$

  – $T \ge 0.25\,\mathrm{s}$:

$$\frac{C_E(\alpha,T)}{C_E(\alpha,t_p)} = \begin{cases} 1/20 & \text{for} \quad t_p < 625\,\mu\mathrm{s} \\ 2\sqrt{t_p} & \text{for} \quad 625\,\mu\mathrm{s} \le t_p < 0.25\,\mathrm{s} \end{cases}$$

FIG. 2. The ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ for $t_{\min} < t_p < 0.25\,\mathrm{s}$ and $t_p < T$. t_p is the duration of a single pulse. T is the exposure duration for a sequence of pulses. α is the apparent source angular subtense in mrad.

$$N < \left(\frac{T}{t_p}\right)^{3/4} \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{32}$$

These limits are plotted as a function of the pulse duration for selected values of the source angular subtense in Fig. 5 for $T = 0.25\,\mathrm{s}$ and in Fig. 6 for $T = 10\,\mathrm{s}$. For a particular pulse duration t_p, if the pulse repetition frequency F or the number of pulses N lies below the appropriate curve for the source subtense α, then rule 1, the single-pulse MPE, will yield the lower limit. If N or F lies on or above the curve, then rule 2, the average power MPE, gives the lower exposure limit. Alternatively, for a particular value for N or F, if the pulse duration t_p lies to the left of the curve, rule 1 yields the lower MPE. If t_p lies on or to the right of the curve, then rule 2 gives the lower MPE.

The limits on the pulse repetition frequency and number of pulses are plotted as a function of the source angular subtense for $T = 0.25\,\mathrm{s}$ in Fig. 7, and for $T = 10\,\mathrm{s}$ in Fig. 8. The pulse repetition frequency limit for pulse durations $t_p < 625\,\mu\mathrm{s}$ can be obtained by scaling the curve for $625\,\mu\mathrm{s}$ by a factor of $(6.25 \times 10^{-4}/t_p)^{3/4}$.

Note that the comparison of rule 2 with rule 1 is of practical importance only when $C_P = 1$. This is true only for $\alpha \le 5\,\mathrm{mrad}$ or $\alpha \ge 100\,\mathrm{mrad}$. For intermediate source sizes, where $C_P < 1$, the comparison between rules 2 and 3 will determine the most restrictive exposure limit.

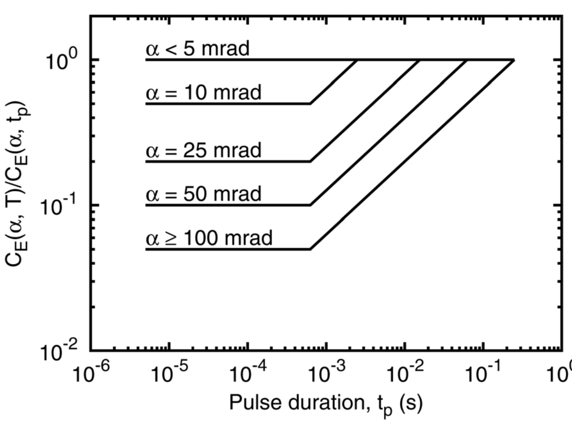

FIG. 3. The ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ for $T \ge 0.25\,\mathrm{s}$ as a function of the pulse duration for several values of the source angular subtense α.

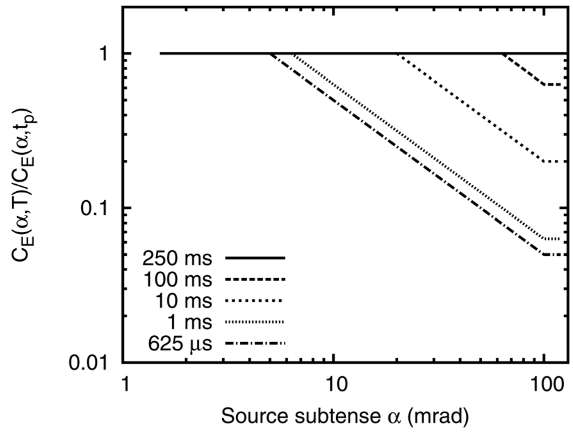

FIG. 4. The ratio $C_E(\alpha,T)/C_E(\alpha,t_p)$ for $T \ge 0.25\,\mathrm{s}$ as a function of the source angular subtense α for selected values of the pulse duration t_p.

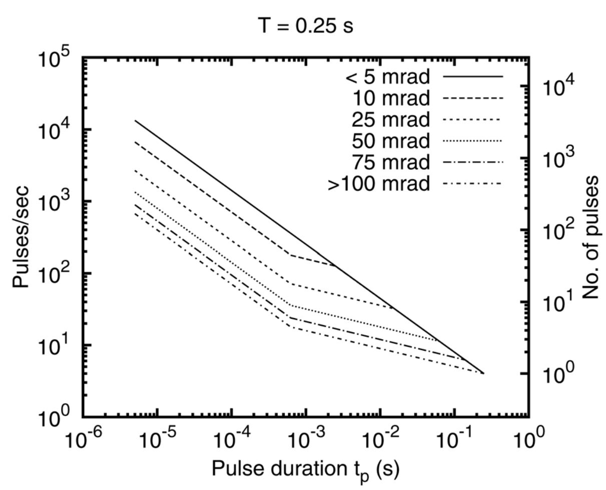

FIG. 5. Maximum pulse rate F and number of pulses N for which rule 1 yields a lower MPE than rule 2 for a total exposure duration $T = 0.25\,\mathrm{s}$ for selected values of the source angular subtense α.

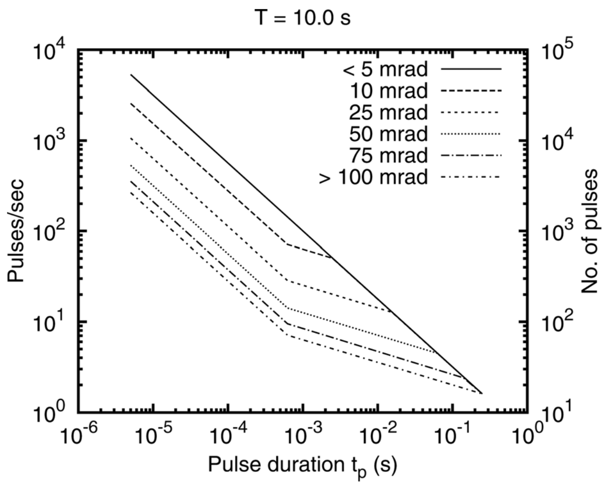

FIG. 6. Maximum pulse rate F and number of pulses N for which rule 1 yields a lower MPE than rule 2 for a total exposure duration $T = 10$ s for selected values of the source angular subtense α.

When $\alpha \leq 5$ mrad, the crossover frequency is obtained by inserting the results of Eq. (27) into Eq. (31):

$$F_{\text{cross}}^{1\to 2} = (t_p^3\, T)^{-1/4} \quad \text{for } \alpha \leq 5\,\text{mrad}. \tag{33}$$

For the total exposure durations of 0.25 and 10 s commonly needed for a safety analysis,

$$F_{\text{cross}}^{1\to 2}(T = 0.25\,\text{s}, \alpha \leq 5\,\text{mrad}) = 1.41\, t_p^{-3/4}, \tag{34}$$

$$F_{\text{cross}}^{1\to 2}(T = 10\,\text{s}, \alpha \leq 5\,\text{mrad}) = 0.56\, t_p^{-3/4}. \tag{35}$$

When $\alpha \geq 100$ mrad, Eqs. (4) and (29) indicate that $C_E(\alpha, T)/C_E(\alpha, t_p) = 2\sqrt{t_p}$ for $t_p \geq 625\,\mu$s. For $t_p < 625\,\mu$s, this ratio is equal to $5/100 = 1/20$. The critical frequency for $\alpha \geq 100$ mrad is

$$F_{\text{cross}}^{1\to 2} = \begin{cases} 0.05\,(t_p^3\, T)^{-1/4} & \text{for} \quad t_p < 625\,\mu\text{s} \\ 2(t_p\, T)^{-1/4} & \text{for} \quad t_p \geq 625\,\mu\text{s} \end{cases} \tag{36}$$

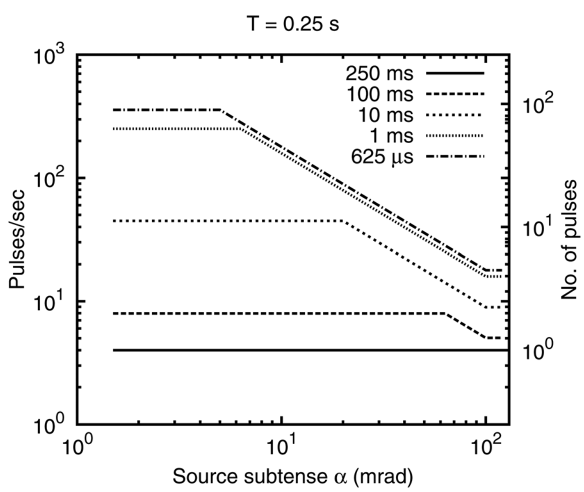

FIG. 7. Maximum pulse rate F and number of pulses N for which rule 1 yields a lower MPE than rule 2 as a function of the source angular subtense. Plot is for total exposure duration $T = 0.25$ s.

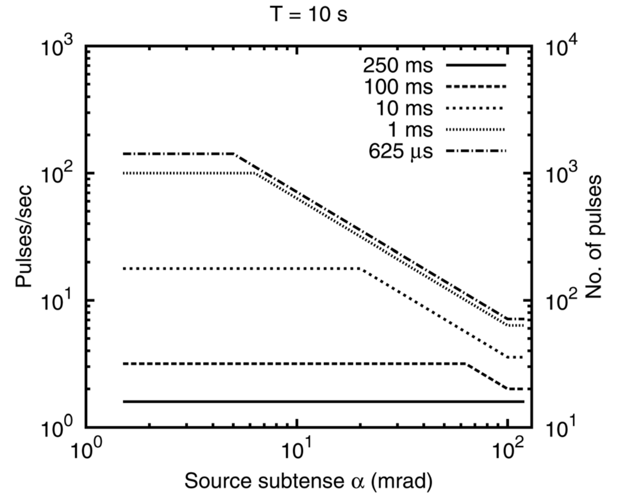

FIG. 8. Maximum pulse rate F and number of pulses N for which rule 1 yields a lower MPE than rule 2 as a function of the source angular subtense. Plot is for total exposure duration $T = 10$ s.

and for exposure durations of 0.25, 10, and 100 s:

$$F_{\text{cross}}^{1\to 2}(T = 0.25\,\text{s}, \alpha \geq 100\,\text{mrad}) = \begin{cases} 0.071\, t_p^{-3/4} & \text{for} \quad t_p < 625\,\mu\text{s} \\ 2.84\, t_p^{-1/4} & \text{for} \quad t_p \geq 625\,\mu\text{s} \end{cases}, \tag{37}$$

$$F_{cross}^{1\to 2}(T = 10\,\text{s}, \alpha \geq 100\,\text{mrad}) = \begin{cases} 0.028\, t_p^{-3/4} & \text{for} \quad t_p < 625\,\mu\text{s} \\ 1.12\, t_p^{-1/4} & \text{for} \quad t_p \geq 625\,\mu\text{s} \end{cases}, \tag{38}$$

$$F_{\text{cross}}^{1\to 2}(T = 100\,\text{s}, \alpha \geq 100\,\text{mrad}) = \begin{cases} 0.016\, t_p^{-3/4} & \text{for} \quad t_p < 625\,\mu\text{s} \\ 0.63\, t_p^{-1/4} & \text{for} \quad t_p \geq 625\,\mu\text{s} \end{cases}. \tag{39}$$

V. CRITERIA FOR RULE 3 < RULE 2

The presence of the multiple pulse correction factor C_P in Eq. (24) complicates the comparison of rules 2 and 3. Using Eq. (21) in Eq. (25) gives an upper limit on the pulse repetition frequency for which rule 3 yields a more restrictive MPE than rule 2. For $T < T_2$, this becomes

$$F < C_P^{-1}\,(t_p^3\, T)^{-1/4}\, \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}. \tag{40}$$

The upper limit on the number of pulses is then (for $T < T_2$)

$$C_P N < \left(\frac{T}{t_p}\right)^{3/4} \frac{C_E(\alpha, T)}{C_E(\alpha, t_p)}, \tag{41}$$

where the factor C_P has been written on the left side of the inequality because it is defined in terms of N. It is easier to work with this equation to determine the upper limit on the number of pulses, then determine the upper limit on the pulse rate using $F = N/T$.

C_P differs from 1.0 only for $5\,\text{mrad} < \alpha \le 100\,\text{mrad}$ (Fig. 1, Eq. (9)). Rule 3 does not need to be evaluated for sources of angular subtense larger or smaller than this range. For sources with extent within this range, the calculation of the maximum number of pulses or pulse repetition frequency for which rule 3 yields the more restrictive MPE is straightforward, but tedious. Only the equations needed to determine the limit on N or F are shown. Furthermore, only total exposure durations $t_\alpha < T \le T_2$ are considered. The specifics of the calculations depend on whether the single-pulse duration t_p is less than $625\,\mu\text{s}$, between $625\,\mu\text{s}$ and t_α, or greater than t_α.

A. $t_{\min} \le t_p < 625\,\mu\text{s}$

For single-pulse duration $t_p < 625\,\mu\text{s}$, the ratio $C_E(\alpha,T)/C_E(\alpha,t_p) = 5/\alpha$ (Fig. 2, Eq. (28d)). Because $t_\alpha \ge 625\,\mu\text{s}$ (Eq. (5)), we have $t_p < t_\alpha$, in which case $\alpha > \alpha_{\max}$. Therefore,

$$C_P = \begin{cases} N^{-1/4} & : N \le 625 \\ 0.2 & : N > 625 \end{cases}. \tag{42}$$

The procedure for determining the upper limit on N and F for which rule 3 provides the lower MPE is a two-step process:

- Step 1: Assume $N^{3\to2}_{\text{cross}} \le 625$ (which implies $F^{3\to2}_{\text{cross}} \le 625/T$). Then, from Eq. (42), $C_P = N^{-1/4}$. Inserting the values for C_P and $C_E(\alpha,T)/C_E(\alpha,t_p)$ in Eq. (41), we calculate

$$N^{3\to2}_{\text{cross}} = \left[\frac{5}{\alpha}\right]^{4/3} \frac{T}{t_p}. \tag{43}$$

If the result is $N^{3\to2}_{\text{cross}} \le 625$, then this is the upper limit on N for which rule 3 is more restrictive than rule 2. The upper limit on the pulse repetition frequency is then found by dividing $N^{3\to2}_{\text{cross}}$ by the exposure duration T:

$$F^{3\to2}_{\text{cross}} = \left[\frac{5}{\alpha}\right]^{4/3} \frac{1}{t_p}. \tag{44}$$

Otherwise, it must be true that $N^{3\to2}_{\text{cross}} > 625$. Proceed to step 2.

- Step 2: For $N^{3\to2}_{\text{cross}} > 625$, Eq. (42) indicates that $C_P = 0.2 = 1/5$. Calculate

$$N^{3\to2}_{\text{cross}} = \frac{25}{\alpha}\left(\frac{T}{t_p}\right)^{3/4}, \tag{45}$$

$$F^{3\to2}_{\text{cross}} = \frac{25}{\alpha}(t_p^3\,T)^{-1/4}. \tag{46}$$

Once $F^{3\to2}_{\text{cross}}$ has been calculated, a comparison with the pulse repetition frequency will indicate which of rule 2 (the average power rule) or rule 3 (the multiple pulse rule) yields the more restrictive exposure limit. If $F < F^{3\to2}_{\text{cross}}$, then rule 3 determines the limit. If $F \ge F^{3\to2}_{\text{cross}}$, rule 2 gives the lower limit.

B. $625\,\mu\text{s} \le t_p < t_\alpha$

In this pulse duration range, $C_E(\alpha,T)/C_E(\alpha,t_p) = \sqrt{t_p/t_\alpha}$ (Fig. 2, Eq. (28d)). Since $t_p < t_\alpha$, we have $\alpha > \alpha_{\max}$. Therefore,

$$C_P = \begin{cases} N^{-1/4} & : N \le 625 \\ 0.2 & : N > 625 \end{cases}. \tag{47}$$

- Step 1: Assume $N^{3\to2}_{\text{cross}} \le 625$ (which implies $F^{3\to2}_{\text{cross}} \le 625/T$). Then $C_P = N^{-1/4}$. Calculate

$$N^{3\to2}_{\text{cross}} = T\,(t_p\,t_\alpha^2)^{-1/3}. \tag{48}$$

If the result is $N^{3\to2}_{\text{cross}} \le 625$, this is the upper limit on N. Then

$$F^{3\to2}_{\text{cross}} = (t_p\,t_\alpha^2)^{-1/3}. \tag{49}$$

Otherwise, it must be true than $N^{3\to2}_{\text{cross}} > 625$. Proceed to step 2.

- Step 2: For $N^{3\to2}_{\text{cross}} > 625$, Eq. (47) indicates that $C_P = 0.2$. Calculate

$$N^{3\to2}_{\text{cross}} = 5\left[\frac{T^3}{t_p\,t_\alpha^2}\right]^{1/4}, \tag{50}$$

$$F^{3\to2}_{\text{cross}} = 5\,(t_p\,T\,t_\alpha^2)^{-1/4}. \tag{51}$$

C. $t_p \ge t_\alpha$

Here, $C_E(\alpha,T)/C_E(\alpha,t_p) = 1$ (Fig. 2, Eq. (28d)). Since $t_p \ge t_\alpha$, we have $\alpha \le \alpha_{\max}$. Therefore,

$$C_P = \begin{cases} N^{-1/4} & : N \le 40 \\ 0.4 & : N > 40 \end{cases}. \tag{52}$$

- Step 1: Assume $N^{3\to2}_{\text{cross}} \le 40$ (which implies $F^{3\to2}_{\text{cross}} \le 40/T$.) From Eq. (52), $C_P = N^{-1/4}$. Calculate

$$N^{3\to2}_{\text{cross}} = T/t_p. \tag{53}$$

If the result is $N^{3\to2}_{\text{cross}} \le 40$, this is the upper limit on N. Then

$$F^{3\to2}_{\text{cross}} = 1/t_p. \tag{54}$$

Note that this condition implies a 100% duty cycle ($\mathcal{D} = F^{3\to2}_{\text{cross}} \times t_p = 1$), i.e., a continuous beam. Thus, for $t_p \ge t_\alpha$ ($\alpha \le \alpha_{\max}$), if it is determined that $N^{3\to2}_{\text{cross}} \le 40$, then rule 3 will always be more restrictive than rule 2. Otherwise, it must be true than $N^{3\to2}_{\text{cross}} > 40$. Proceed to step 2.

- Step 2: For $N^{3\to2}_{\text{cross}} > 40$, Eq. (52) indicates that $C_P = 0.4$. Calculate

$$N^{3\to2}_{\text{cross}} = \frac{5}{2}\left(\frac{T}{t_p}\right)^{3/4}, \tag{55}$$

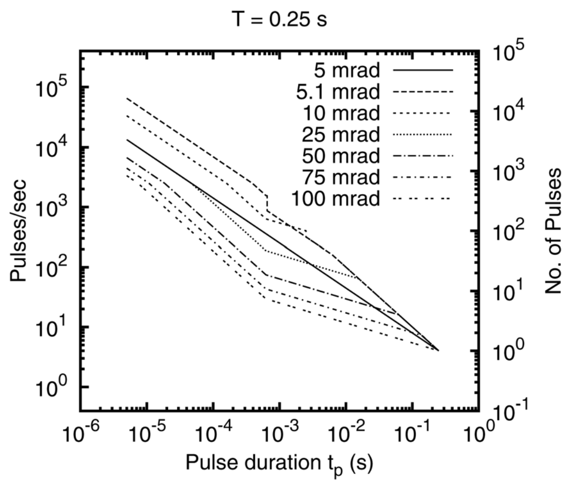

FIG. 9. Maximum pulse rate F and number of pulses N for which rule 3 yields a lower MPE than rule 2 for an exposure duration $T=0.25$ s for selected values of the source angular subtense α.

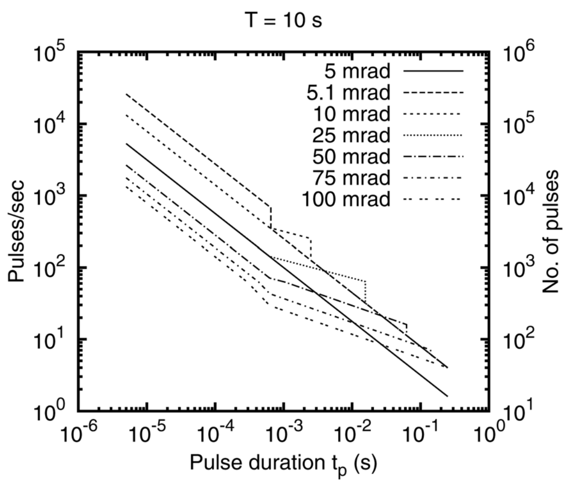

FIG. 10. Maximum pulse rate F and number of pulses N for which rule 3 yields a lower MPE than rule 2 for an exposure duration of $T=10$ s for selected values of the source angular subtense α.

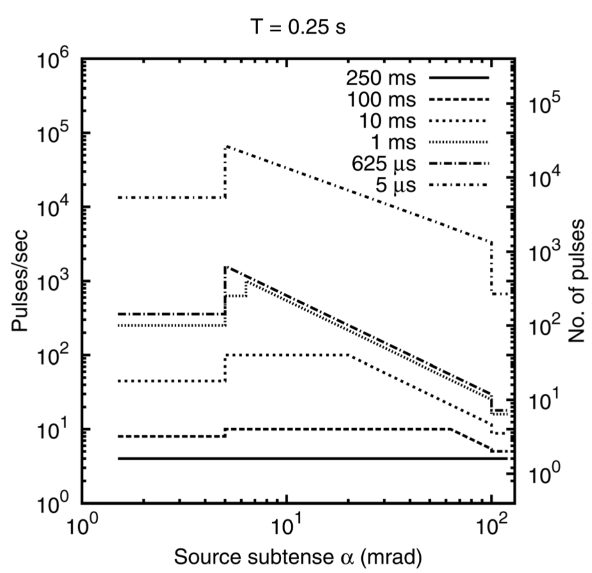

FIG. 11. Maximum pulse rate F and number of pulses N for which rule 3 yields a lower MPE than rule 2 for an exposure duration $T=0.25$ s as a function of source angular subtense for selected pulse durations.

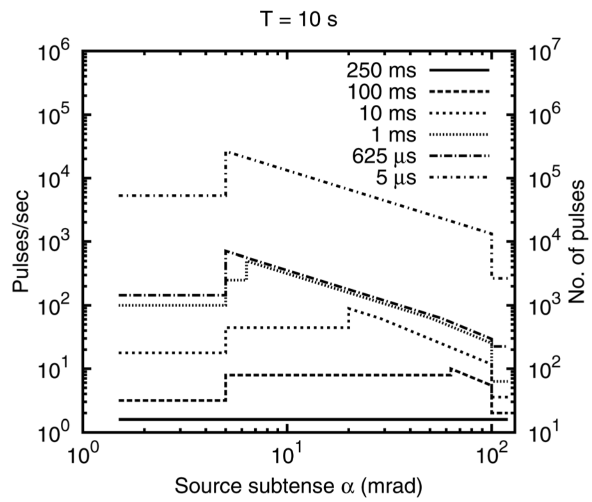

FIG. 12. Maximum pulse rate F and number of pulses N for which rule 3 yields a lower MPE than rule 2 for an exposure duration $T=10$ s as a function of source angular subtense for selected pulse durations.

$$F_{\text{cross}}^{3\to 2} = \frac{5}{2}\left(T\, t_p^3\right)^{-1/4}. \tag{56}$$

D. Results

The limits on the pulse repetition frequency and number of pulses for which rule 3 yields a lower MPE than rule 2 are plotted in Fig. 9 for $T=0.25$ s and in Fig. 10 for $T=10$ s.

These limits are plotted as a function of the source angular subtense in Fig. 11 for $T=0.25$ s and Fig. 12 for $T=10$ s.

VI. SUMMARY

In the upcoming revisions to the ANSI, ICNIRP, and IEC laser safety standards, the time-dependence of α_{max} as well as the amended multiple pulse exposure analysis rules have resulted in substantial changes to the maximum permissible exposure limit, for small as well as extended sources. For small sources, with angular subtense α smaller than 5 mrad, the analysis has been simplified in comparison to current guidelines and standards because there is no need to explicitly consider a multiple pulse correction factor. However, for extended sources, a safety analysis may be complex.

We have presented criteria for the critical multiple pulse rule which can help to simplify a safety analysis, and to understand the impact of the changes on permitted emission levels of laser products. For BJL: The opinions or assertions contained herein are the private views of the authors and are no to be construed as official or as reflecting the views of the Department of the Army or the Department of Defense.

[1]American National Standards Institute (ANSI), *American National Standard for Safe Use of Lasers* (Laser Institute of America, Orlando, FL, 2007).
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